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Álgebra

Tema: Números complejos II

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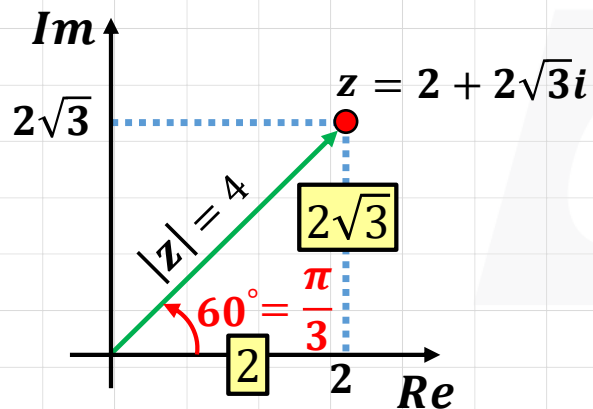
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1. Dados los siguientes complejos $z = 2 + 2\sqrt{3}i$ y $w = -1 - i$. Calcule el complejo $z \times w$ en su forma polar.

Resolución:

Expresamos cada número complejo en su forma polar

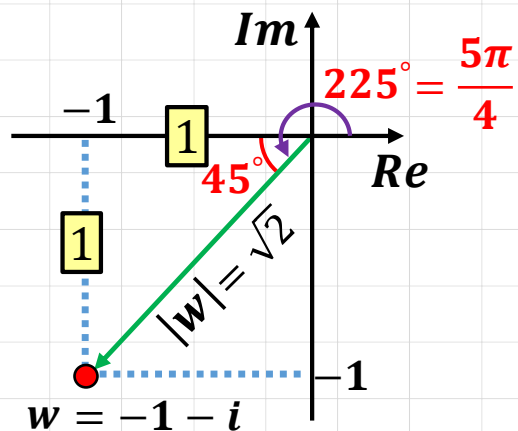
$$z = 2 + 2\sqrt{3}i$$



$$z = 4 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$z = 4 \operatorname{cis} \frac{\pi}{3} = 4e^{i\frac{\pi}{3}}$$

$$w = -1 - i$$



$$w = \sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$$

$$w = \sqrt{2} \operatorname{cis} \frac{5\pi}{4} = \sqrt{2}e^{i\frac{5\pi}{4}}$$

Piden

$$z \times w = \left(4 \operatorname{cis} \frac{\pi}{3} \right) \left(\sqrt{2} \operatorname{cis} \frac{5\pi}{4} \right)$$

$$= 4\sqrt{2} \operatorname{cis} \left(\frac{\pi}{3} + \frac{5\pi}{4} \right)$$

$$= 4\sqrt{2} \operatorname{cis} \left(\frac{19\pi}{12} \right) \quad \text{Clave B}$$

Recordar:

Sean los números complejos z y w no nulos.

$$z \times w = |z| \cdot |w| \operatorname{cis}(\theta + \alpha)$$

2. Calcule el módulo de z si $z^*(\operatorname{sen}35^\circ + i\cos35^\circ) = \frac{\left(3 + 4\left(\frac{1-i}{1+i}\right)\right)(6 + \sqrt{7}i)}{(6 - \sqrt{7}i)\left(5\operatorname{cis}\frac{\pi}{8}\right)}$

Resolución:

$$\text{En } z^*(\operatorname{sen}35^\circ + i\cos35^\circ) = \frac{\left(3 + 4\left(\frac{1-i}{1+i}\right)\right)(6 + \sqrt{7}i)}{(6 - \sqrt{7}i)\left(5\operatorname{cis}\frac{\pi}{8}\right)}$$

$$|z^*(\operatorname{sen}35^\circ + i\cos35^\circ)| = \left| \frac{(3 - 4i)(6 + \sqrt{7}i)}{(6 - \sqrt{7}i)\left(5\operatorname{cis}\frac{\pi}{8}\right)} \right|$$

$$|z^*| \cdot |\operatorname{sen}35^\circ + i\cos35^\circ| = \left| \frac{(3 - 4i)(6 + \sqrt{7}i)}{(6 - \sqrt{7}i)\left(5\operatorname{cis}\frac{\pi}{8}\right)} \right|$$

$$|z| \cdot 1 = \frac{|3 - 4i| \cdot |6 + \sqrt{7}i|}{|6 - \sqrt{7}i| \cdot |5\operatorname{cis}\frac{\pi}{8}|}$$

$$|z| = \frac{5}{5} = 1$$

Clave A

Recordar:

Sea $z, w \in \mathbb{C}$

$$|z| = |\bar{z}| = |z^*|$$

$$|z \cdot w| = |z||w|$$

$$\left| \frac{z}{w} \right| = \frac{|z|}{|w|}$$

$$z = |z| \cdot \operatorname{cis}\theta$$

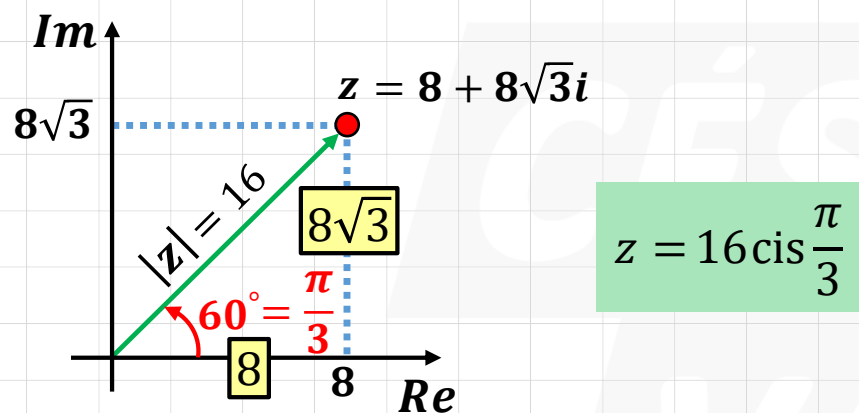
$$|\cos\theta + i\operatorname{sen}\theta| = 1$$

$$|\operatorname{sen}\theta + i\cos\theta| = 1$$

3. Determine una raíz cuarta que se ubica en el tercer cuadrante de $z = 8 + 8\sqrt{3}i$.

Resolución:

Llevamos $z = 8 + 8\sqrt{3}i$ a la forma cis.



Hallamos la raíz cuarta de $z = 16\text{cis}\frac{\pi}{3}$

$$\sqrt[4]{z} = \sqrt[4]{16\text{cis}\left(\frac{\pi}{3}\right)} = \sqrt[4]{16} \cdot \sqrt[4]{\text{cis}\left(\frac{\pi}{3}\right)}$$

$$\sqrt[4]{z} = 2\text{cis}\left(\frac{\frac{\pi}{3} + 2k\pi}{4}\right) = \begin{cases} k=0; z_0 = 2\text{cis}\left(\frac{\pi}{12}\right) \\ k=1; z_1 = 2\text{cis}\left(\frac{7\pi}{12}\right) \\ k=2; z_2 = 2\text{cis}\left(\frac{13\pi}{12}\right) \\ k=3; z_3 = 2\text{cis}\left(\frac{19\pi}{12}\right) \end{cases}$$

donde $k = 0, 1, 2, 3$.

$\pi < \theta = \frac{13\pi}{12} < \frac{3\pi}{2}$

Piden la raíz cuarta que se ubique en el tercer cuadrante

$$2\text{cis}\left(\frac{13\pi}{12}\right)$$

Recordar:

$$\sqrt[n]{z} = \sqrt[n]{|z|} \text{cis}\left(\frac{\theta + 2k\pi}{n}\right)$$

donde $k = 0, 1, 2, \dots, n - 1$.

Clave D

4. Calcule el menor valor entero de tres cifras que puede tomar n para que se cumpla $\left(\operatorname{cis} \frac{\pi}{12}\right)^n = \frac{1+i}{\sqrt{2}}$

Resolución:

Sea $z = \frac{1+i}{\sqrt{2}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$, lo llevamos a la forma **cis**.

$$|z| = 1 \quad \theta = \frac{\pi}{4}$$

$$\Rightarrow z = \operatorname{cis} \left(\frac{\pi}{4} \right)$$

Como $\left(\operatorname{cis} \frac{\pi}{12}\right)^n = \frac{1+i}{\sqrt{2}}$ (Aplicamos De Moivre)

$$\operatorname{cis} \left(\frac{n\pi}{12} \right) = \operatorname{cis} \left(\frac{\pi}{4} \right) \quad (\text{Ecuación trigonométrica})$$

$$\Rightarrow \frac{n\pi}{12} = \frac{\pi}{4} + 2k\pi ; k \in \mathbb{Z}$$

$$\frac{n}{12} = \frac{1}{4} + 2k \quad (\text{Por 12})$$

$$n = 3 + 24k ; k \in \mathbb{Z}$$

$$\text{Si } k = 4 \Rightarrow n = 99$$

$$\text{Si } k = 5 \Rightarrow n = 123 \quad (\text{Menor valor de 3 cifras})$$

$$\text{Si } k = 6 \Rightarrow n = 147$$

⋮

Clave C

Recordar:

$$(\operatorname{cis} \theta)^n = \operatorname{cis}(\theta n)$$

5. Si $z \in \mathbb{C}$ y se cumple que $z - \frac{1}{z} = 2i \operatorname{sen} \theta$; $z \neq 0$. Calcule $z^n - z^{-n}$.

Resolución:

Como $z \in \mathbb{C} \wedge z \neq 0$

$$\Rightarrow \begin{aligned} z &= |z|(\cos \theta + i \operatorname{sen} \theta) \\ \frac{1}{z} &= \frac{1}{|z|}(\cos \theta - i \operatorname{sen} \theta) \end{aligned}$$

$$z - \frac{1}{z} = \cos \theta \left(|z| - \frac{1}{|z|} \right) + i \operatorname{sen} \theta \left(|z| + \frac{1}{|z|} \right)$$

$$z - \frac{1}{z} = 2i \operatorname{sen} \theta \text{ (Dato)}$$

$$\Rightarrow |z| + \frac{1}{|z|} = 2 \Rightarrow |z| = 1$$

$$\Rightarrow z = (\cos \theta + i \operatorname{sen} \theta) \wedge \frac{1}{z} = (\cos \theta - i \operatorname{sen} \theta)$$

$$z^n = (\cos \theta + i \operatorname{sen} \theta)^n$$

$$z^n = \cos(n\theta) + i \operatorname{sen}(n\theta)$$

$$\left(\frac{1}{z} \right)^n = (\cos \theta - i \operatorname{sen} \theta)^n$$

$$z^{-n} = \cos(n\theta) - i \operatorname{sen}(n\theta)$$

$$z^n - z^{-n} = 2i \operatorname{sen}(n\theta)$$

Clave A

Recordar:

$$z = |z|(\cos \theta + i \operatorname{sen} \theta)$$

$$z^n = |z|^n [\cos(n\theta) + i \operatorname{sen}(n\theta)]; n \in \mathbb{N}$$

$$\frac{1}{z} = \frac{1}{|z|}(\cos \theta - i \operatorname{sen} \theta)$$

6. Entre los números complejos z que satisfacen la condición $|z - 50i| \leq 40$. Halle el número complejo con menor argumento.

Resolución:

Sea $z = x + yi$

Graficamos $|z - 50i| \leq 40$

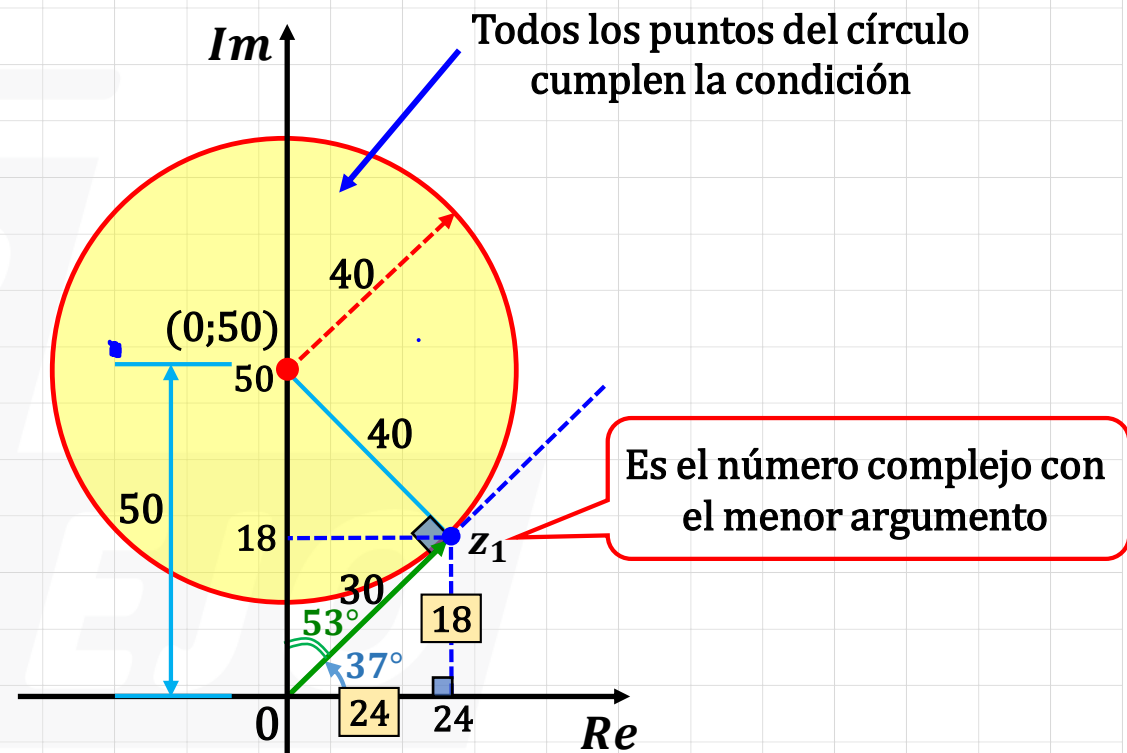
$$|x + yi - 50i| \leq 40$$

$$|x + (y - 50)i| \leq 40$$

$$\sqrt{x^2 + (y - 50)^2} \leq 40 \text{ (Elevamos al cuadrado)}$$

$$x^2 + (y - 50)^2 \leq 40^2$$

Representa un círculo con centro en $(0 ; 50)$ y radio 40



Del gráfico

$$z_1 = 24 + 18i$$

Clave B

7. Indique la secuencia correcta de verdad (V) o falsedad (F) según corresponda.

I. $i^i = e^{-\frac{\pi}{2}}$

II. e^{3+2i} , tiene argumento 2 y módulo 1

III. El complejo $-3e^{\frac{\pi}{5}i}$, tiene módulo -3

Resolución:

I. $i^i = e^{-\frac{\pi}{2}}$ (V)

$$i^i = \left(e^{\frac{\pi}{2}i}\right)^i = e^{\frac{\pi}{2}i^2} = e^{-\frac{\pi}{2}}$$

II. e^{3+2i} , tiene argumento 2 y módulo 1 (F)

$$e^{3+2i} = e^3 \cdot e^{2i}$$

Argumento: 2

Módulo: e^3

III. El complejo $-3e^{\frac{\pi}{5}i}$, tiene módulo -3 (F)

El módulo jamás es **NEGATIVO**.



VFF

Clave C

Recordar:

$$i = \text{cis}\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}i}$$

$$|z| \geq 0$$

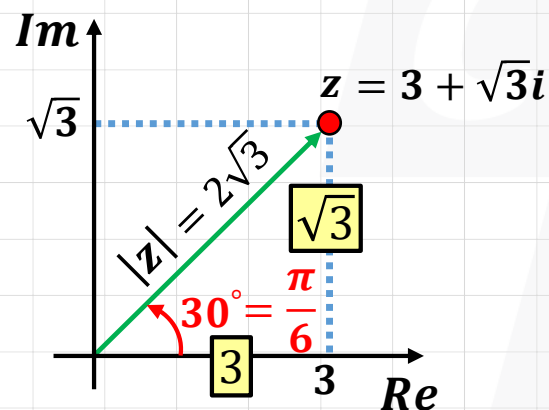
8. Si $z_1 = 3 + \sqrt{3}i$, $z_2 = -2\text{cis}\left(\frac{\pi}{5}\right)$, $z_3 = 7e^{\frac{\pi}{4}i}$. Calcule el argumento de $z_4 \in \mathbb{C}$ si

$$z_1^6 \cdot z_2^{10} = z_3^4 \cdot z_4^7$$

Resolución:

Hallamos los argumentos de z_1 y z_2

$$z_1 = 3 + \sqrt{3}i$$



$$z_2 = -2\text{cis}\left(\frac{\pi}{5}\right)$$

$$z_2 = 2(-1)\text{cis}\left(\frac{\pi}{5}\right)$$

$$z_2 = 2\text{cis}(\pi)\text{cis}\left(\frac{\pi}{5}\right) = 2\text{cis}\left(\pi + \frac{\pi}{5}\right) = 2\text{cis}\left(\frac{6\pi}{5}\right)$$

$$\text{Como } z_1^6 \cdot z_2^{10} = z_3^4 \cdot z_4^7$$

$$\text{Arg}(z_1^6 \cdot z_2^{10}) = \text{Arg}(z_3^4 \cdot z_4^7)$$

$$\text{Arg}(z_1^6) + \text{Arg}(z_2^{10}) = \text{Arg}(z_3^4) + \text{Arg}(z_4^7)$$

$$6\text{Arg}(z_1) + 10\text{Arg}(z_2) = 4\text{Arg}(z_3) + 7\text{Arg}(z_4)$$

$$6\left(\frac{\pi}{6}\right) + 10\left(\frac{6\pi}{5}\right) = 4\left(\frac{\pi}{4}\right) + 7\text{Arg}(z_4)$$

$$\cancel{\pi} + 12\pi = \cancel{\pi} + 7\text{Arg}(z_4)$$

$$\frac{12\pi}{7} = \text{Arg}(z_4) \quad \text{Clave A}$$

Recordar:

$$\text{Arg}(z \cdot w) = \text{Arg}(z) + \text{Arg}(w)$$

$$\text{Arg}(z^n) = n \cdot \text{Arg}(z)$$

$$-1 = \text{cis}(\pi)$$

9. Determine la parte imaginaria de $z = e^{i \cos \theta}$; $0 < \theta < \frac{\pi}{2}$

Resolución:

$$\text{En } z = e^{i \cos \theta}$$

$$z = e^{i(\cos \theta + i \sin \theta)}$$

$$z = e^{i \cos \theta + i^2 \sin \theta}$$

$$z = e^{i \cos \theta - \sin \theta}$$

$$z = e^{-\sin \theta + i \cos \theta}$$

$$z = e^{-\sin \theta} \cdot e^{i \cos \theta}$$

$$z = e^{-\sin \theta} (\cos(\cos \theta) + i \sin(\cos \theta))$$

$$z = e^{-\sin \theta} \cdot \cos(\cos \theta) + i e^{-\sin \theta} \cdot \sin(\cos \theta)$$

$$\text{Piden la } \text{Im}(z) = e^{-\sin \theta} \cdot \sin(\cos \theta)$$

Clave C

$$e^{i\psi} = \cos \psi + i \sin \psi$$

$$e^{i \cos \theta} = \cos(\cos \theta) + i \sin(\cos \theta)$$

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